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292. Proposed by C. N. SCHMALL, New York City.

In a bombardment, a battleship directs its fire at a fort standing on a hill whose height is a feet above the sea level. The angle of elevation of the fort is found to be ϕ. If the initial velocity of the projectile is v, show that the fort will *not* be struck if $v < \sqrt{ag(1 + \csc \phi)}$.

NUMBER THEORY.

When this issue was made up, solutions had been received for 200, 203, 206, 207 and 210. Solutions of 189, 191, 192, 196, 202, 204–5, and 208–9 are desired.

211. Proposed by E. T. BELL, Seattle, Washington.

If an odd perfect number exists, the total number of its divisors is a multiple of 2, but not of 4; or, what is the same thing, an odd perfect number must be of the form $p^{2a-1}n^2$, where p is prime and a is odd.

212. Proposed by ELMER SCHUYLER, Brooklyn, New York.

Given any positive integer N greater than 1; to prove that the sum of all the positive integers less than N and relatively prime to N equals $\frac{1}{2}N \cdot \varphi(N)$.

213. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that no relatively prime integers x and y exist such that the difference of their fourth powers is a cube.

214. Proposed by A. J. KEMPNER, University of Illinois.

Let a be a positive integer $\geqq 2$, and let $T(n)$ denote the number of distinct divisors of the positive integer n, including both 1 and n, so that $T(1) = 1$, $T(2) = 2$, $T(3) = 2$, $T(4) = 3$, ... Show that

$$\Sigma_{n=1}^{n=\infty} T(n)/a^n = \Sigma_{n=1}^{n=\infty} 1/(a^n - 1).$$

The special case $a = 10$ gives, as is easily seen:

$$9 \sum_{n=1}^{n=\infty} \frac{T(n)}{10^n} = \frac{1}{1} + \frac{1}{11} + \frac{1}{111} + \frac{1}{1111} + \cdots.$$

SOLUTIONS OF PROBLEMS.

ALGEBRA.

396. Proposed by H. E. TREFETHEN, Colby College.

Show that $1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots = \sqrt{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$.

I. Solution by Horace Olson, Chicago, Illinois.

Let S_1 represent the sum of the series, $x + \frac{x^3}{3} - \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} + \frac{x^{11}}{11} - \cdots$, of which the first member of the proposed equation is a particular case. Let S_2 represent the sum of the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \frac{x^{11}}{11} + \cdots$. By calculus,

$$S_1 = \int_0^x (1 + x^2 - x^4 - x^6 + x^8 + x^{10} - \cdots)dx = \int_0^x \left(\frac{1 + x^2}{1 + x^4}\right) dx$$

$$= \frac{\tan^{-1}(\sqrt{2}x + 1) + \tan^{-1}(\sqrt{2}x - 1)}{\sqrt{2}}$$

Also,

$$S_2 = \int_0^x (1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \cdots)dx = \int_0^x \left(\frac{1}{1+x^2}\right) dx = \tan^{-1} x.$$

Letting $x = 1$, we find

$$S_1 = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots = \frac{\tan^{-1}(\sqrt{2}+1) + \tan^{-1}(\sqrt{2}-1)}{\sqrt{2}}$$

$$= \frac{\tan^{-1}(\sqrt{2}+1) + \cot^{-1}(\sqrt{2}+1)}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}.$$

Similarly, we find for $x = 1$,

$$S_2 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots = \tan^{-1} 1 = \frac{\pi}{4}.$$

Therefore

$$\frac{\pi}{2\sqrt{2}} = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots = \sqrt{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots\right).$$

II. Solution by Hermon L. Slobin, University of Minnesota.

If we develop $f(x) = 1$ as a Fourier Series

$$f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_m \sin mx + \cdots,$$

where

$$a_m = \frac{2}{\pi}\int_0^\pi f(x) \sin mx dx,$$

we note that

$$\int_0^\pi \sin mx dx = \frac{1}{m}(1 - \cos m\pi) = 0 \quad \text{or} \quad \frac{2}{m},$$

according as m is even or odd. Hence,

$$1 = \frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right).$$

Letting $x = \pi/4$, we have

$$\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{3} - \frac{\sqrt{2}}{2}\cdot\frac{1}{5} - \frac{\sqrt{2}}{2}\cdot\frac{1}{7} + \cdots\right) = \frac{\sqrt{2}}{2}\left(1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \cdots\right).$$

Therefore, the left member of the proposed identity $= \sqrt{2} \cdot \pi/4$.

But we note that, since

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots,$$

upon putting $x = 1$, we have $\pi/4 = 1 - 1/3 + 1/5 - \cdots$ which is the series in the right member of the proposed identity. Hence

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\cdots=\sqrt{2}\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\right).$$

III. Solution by J. W. Campbell, University of Chicago.

Let S = sum of the series,

$$\sin 2\theta-\frac{1}{2}\sin 4\theta+\frac{1}{3}\sin 6\theta-\cdots.$$

Then S is the coefficient of i in $e^{2i\theta}-\frac{1}{2}e^{4i\theta}+\frac{1}{3}e^{6i\theta}-\cdots$, that is, in $\log_e(e^{2i\theta}+1)$, or in $\log_e e^{i\theta}(e^{i\theta}+e^{-i\theta})$, or finally in $i\theta+\log_e(2\cos\theta)$, which is θ itself.

Take $\theta=\pi/8$. Then

$$\frac{\pi}{8}=\frac{1}{\sqrt{2}}-\frac{1}{2}+\frac{1}{3\sqrt{2}}+0-\frac{1}{5\sqrt{2}}+\frac{1}{6}\cdots$$

$$=\frac{1}{\sqrt{2}}\left(1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}\cdots\right)-\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{5}\cdots\right).$$

That is,

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\cdots=\sqrt{2}\left[\frac{\pi}{8}+\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{5}\cdots\right)\right]. \qquad (1)$$

By Gregory's Series (Loney's *Trigonometry*, Part II, § 94),

$$\theta=\tan\theta-\frac{1}{3}\tan^3\theta+\frac{1}{5}\tan^5\theta-\cdots,\quad\left(-\frac{\pi}{4}\leqq\theta\leqq\frac{\pi}{4}\right).$$

Take $\theta=\pi/4$. Then $\pi/4=(1-1/3+1/5-\cdots)$. Hence,

$$\frac{\pi}{8}=\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{5}-\cdots\right). \qquad (2)$$

Therefore, from (1) and (2),

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\cdots=\sqrt{2}\left(1-\frac{1}{3}+\frac{1}{5}-\cdots\right).$$

Also solved by C. E. Horne, A. L. McCarty, A. M. Harding, David F. Kelley, Elmer Schuyler, C. N. Schmall, and J. W. Clawson.

ALGEBRA.

397. Proposed by W. H. BUSSEY, University of Minnesota.

12 oxen are turned into a pasture of $3\frac{1}{3}$ acres and eat all the grass in 4 weeks so that the pasture is bare. 21 oxen are turned into a pasture of 10 acres and eat all the grass in 9 weeks. How many oxen will eat all the grass of 24 acres in just exactly 18 weeks, it being assumed that the grass in all the pastures is at the same height when the oxen are turned in, and that the grass grows at a uniform rate.